The Effect of Volume Relaxation on Elastohydrodynamic Lubrication

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In elastohydrodynamic lubrication (EHD) three important non-Newtonian effects arise. These are volume viscoelasticity, shear viscoelasticity, and the variation of viscosity with shear rate. All these effects tend to decrease the shear stress or traction.

In this paper the effect of volume relaxation on EHD is examined using experimental viscosity data obtained in a simple viscometric flow. It is shown that the viscosity of a fluid during EHD is unlikely to reach its equilibrium value. Approximations to the viscosity as a function of time lead to the conclusion that volume and shear viscoelasticity have effects which are of the same order of magnitude and will be difficult to separate except by an exact knowledge of the shear rate and pressure profiles.

NOTATION

De	Deborah Number
G	shear modulus
p	pressure
p_0	constant in Van der Waal's equation
R	gas constant
T	temperature or residence time
t	time
v	volume
v_0	Van der Waals volume
$\dot{\gamma}$	shear rate
τ	shear stress
$\mu(T)$	viscosity as a function of temperature

INTRODUCTION

Elastohydrodynamic lubrication (EHD) is one of the most important lubrication regimes, as it determines the upper limit of practical operation of a rolling bearing. EHD is characterized by the fact that large surface deformation of the bearing surface takes place. Under extreme conditions the pressure profile approaches that of a dry static contact, even though there remains a layer of fluid between the solid bearing surfaces. In such situations the peak pressure can be as high as 3×10^9 Pa. To maintain effective lubrication under such conditions the rotational speed of the bearing has to be high, and it is well known that in such cases lubricants do not behave as Newtonian fluids.

Three different non-Newtonian effects have been considered. Crook **(1)**, Johnson and Roberts **(2)**, etc. consider that the major cause of the non-Newtonian behaviour is shear viscoelasticity as the fluid enters the contact region, as there is an abrupt change in shear rate at this point. From the geometry of an EHD contact it would appear that the shear rate profile leads the pressure profile, and is constant before the pressure starts to rise. The relaxation time of the lubricant outside the high pressure region will be about 10^{-10} s which is almost certainly shorter than the time between the shear and pressure profiles starting to rise. Later it will be shown that even if the pressure profile leads the shear rate profile the shear stress is altered only by a factor of about two.

Hirst and Moore **(3)** have put forward the view that the drop in traction is due to the lubricant acting as an Eyring fluid with a viscosity given by $\mu(\tau) = \mu(0)$ $a\tau/\sinh(a\tau)$ rather than the Newtonian value $\mu(0)$. This almost certainly plays an important role as the stresses should be large enough to produce such effects.

The third suggestion, which has received little attention, is that the lubricant volume does not have sufficient time to respond to the pressure change and consequently the viscosity is below its equilibrium value **(4)** **(5)**. Such retarded volume responses have been modelled using a Kelvin–Voigt spring and dashpot mechanical analogue and has gained the name volume viscoelasticity. These effects are well known in polymer systems at temperatures near and below their glass transition temperature (T_g). Kovacs **(6)** has shown that for quite large temperature steps the volume variation is proportional to the logarithm of the time, over a large part of the relaxation. This behaviour can also be observed in lubricants below their glass transition temperature **(7)**. Free volume theory as used by Kovacs suggests that this behaviour occurs at all temperatures but cannot be observed elsewhere as the material relaxation time is incompatible with experimental techniques. At temperatures encountered in lubrication mechanics the material relaxation time at atmospheric pressure is likely to be less than 10^{-9} s and so cannot be observed in simple dilatometry experiments. To infer what does occur at high temperatures it is possible to use the Deborah Number **(8)**. This has been used previously to look at time scales inaccessible to simple experiments **(9)**, but only for situations where the viscosity is constant. In this paper the implications of volume relaxation are examined in the context of lubrication mechanics. Experimental viscosity results obtained during volume relaxation below T_g are presented in a non-dimensional form and are compared with the behaviour expected in EHD with the aid of the Deborah Number. This comparison shows that during EHD the fluid is unlikely to be at its equilibrium volume or viscosity.

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THEORY

The response of a lubricant to a step change in temperature is similar to that observed for polymers with the volume varying linearly with the logarithm of the time over a large part of the relaxation. It is assumed that the lubricant responds to the temperature step as if no instantaneous volume change occurred, thus forcing the internal pressure to change. Using an equation of state of the form $(p + p_0)(v - v_0) = RT$ gives the change in pressure in the lubricant as

$$\Delta p = R\Delta T/(v - v_0)$$

The response to a pressure step is assumed to be an initial adiabatic response to a new temperature and the applied pressure. This is then followed by a thermal equilibriation back to the ambient temperature. This is similar to the temperature step discussed above, changing the pressure in the liquid. After this the pressure is slowly returned to the applied pressure by volume relaxation. From the equation of state the pressure and volume during relaxation are related by $p = -RT\,\mathrm{d}v/(v - v_0)^2$; this entire process is shown in Fig. 1 which is linear for small changes in volume $\{\mathrm{d}v \ll (v - v_0)\}$. In this equation Δp has been replaced by p as the applied hydrostatic pressure before the step is small compared with the changes considered by the two equations. This equation can now be used to link the viscosity–pressure relation with the volume variation with time during the relaxation process.

For small pressure variations, the viscosity can be regarded as exponentially dependent on the pressure, that is, $\mu(p, T) = \mu(T) \exp(bp)$. Writing the volume variation as $\mathrm{d}v = m - n \ln(t)$ and substituting into the viscosity–pressure relation via the equation of state gives

$$\mu(t) = \mu(T) \exp[-ab\{m - n \ln(t)\}] \tag{1}$$

where $a = RT/(v - v_0)^2$.

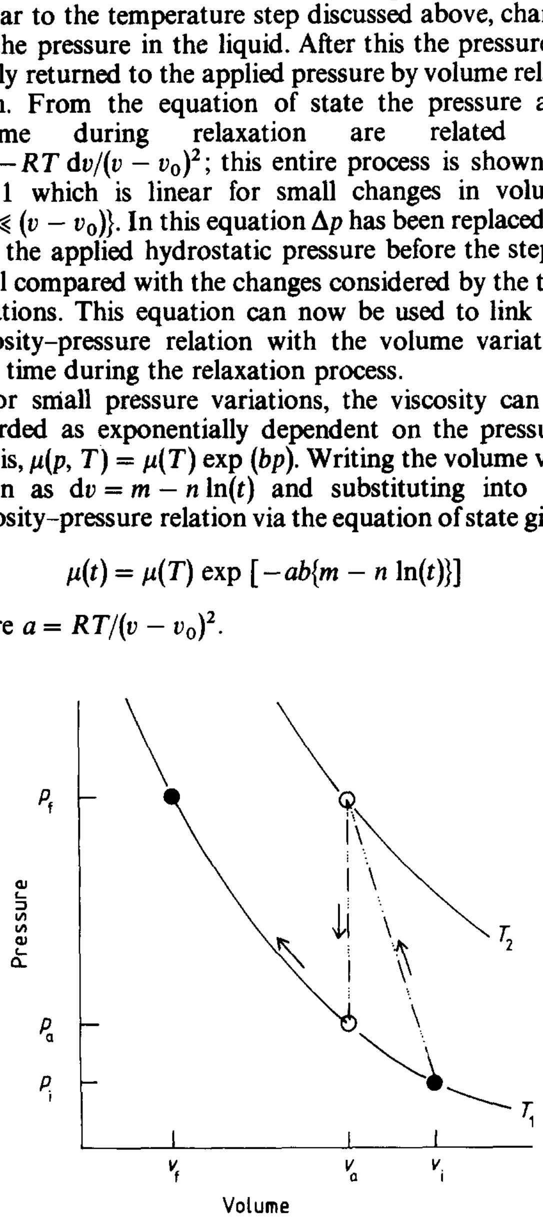

Fig. 1. The idealized path of a material in response to a change of pressure from p_i to p_f. $(p_i, v_i) - (p_f, v_a)$ is the adiabatic response which changes the temperature from the equilibrium temperature T_1 to some temperature T_2, $(p_f, v_a) - (p_a, v_a)$ is the thermal equilibriation with no associated volume change, and $(p_a, v_a) - (p_f, v_f)$ is the isothermal volume change

Rearranging this gives

$$\mu(t)/\mu(T) = \exp(-abm)t^{abn} \tag{2}$$

where $\exp(-abm) = \{\exp(m/n)\}^{-abn}$.
m/n gives the intercept on the ln(time) axis, which is expected to be approximately equal to the Maxwell relaxation time τ_M(7). The Maxwell relaxation time is given by $\mu(T)/G$, where G is the shear rigidity. This gives the result

$$\mu(t)/\mu(T) = A(t/\tau_M)^{abn} \tag{3}$$

where the constant of proportionality A has been included to allow for the possibility that τ_M is not exactly equal to $\exp(m/n)$. Experiments over different time scales can now be compared using the Deborah Number defined by $De = \tau_M/T_e$ where T_e is the experimental time. Substituting this into equation (3) and putting $\xi = abn$ gives

$$\mu(t)/\mu(T) = A(t/T_e De)^{\xi} \tag{4}$$

The Deborah Number has frequently been used, but only for situations where the viscosity and hence the relaxation time has been constant over the time of the experiment.

EXPERIMENTAL

The viscosity results reported here form part of a study to find the path of a fluid element in p-v-T space as a result of a step change in temperature. The viscosities of a synthetic oil Bis m-m-phenoxy phenoxy phenyl ether (OS 138), and polystyrene 580 MW were monitored over the transient responses to various negative step changes in temperature. The temperature steps were made from $(T_g + 1)$°C to below the glass transition temperature. The temperature change took about 200 s to complete. The shortest transient reported here corresponds to a 5°C drop in temperature and had a duration of over 10^5 s measured dilatometrically (7) (the temperature change during dilatometric measurements took only about 30–40 s). Thus for the viscometric results the temperature change occupied less than 0·2 per cent of the transient response time and it was felt that it could be neglected. The viscometer used was a parallel plate torsional viscometer with a magnetic suspension and drag cup motor, similar to that described by Plazek (10). The liquids were held at constant stress prior to the temperature change for sufficient time to allow the material to reach steady state flow, thus ensuring that no shear viscoelastic effects are present in the reported results.

RESULTS AND DISCUSSION

Four transient responses were observed for each of the liquids used in the study. The smallest temperature step was 5°C and the largest was 13°C (from −19°C to −32°C). Over the temperature range of these steps the viscosity functions of the two liquids are similar, ranging from about 10^{14}–10^{10} Pa s. At such high viscosities the volume relaxation is slow, and its measurement presents no real problems. Not all the transients were followed to completion as for the larger steps the transient time would have been prohibitively long, estimated as 10^7 s (4 months) for the biggest step.

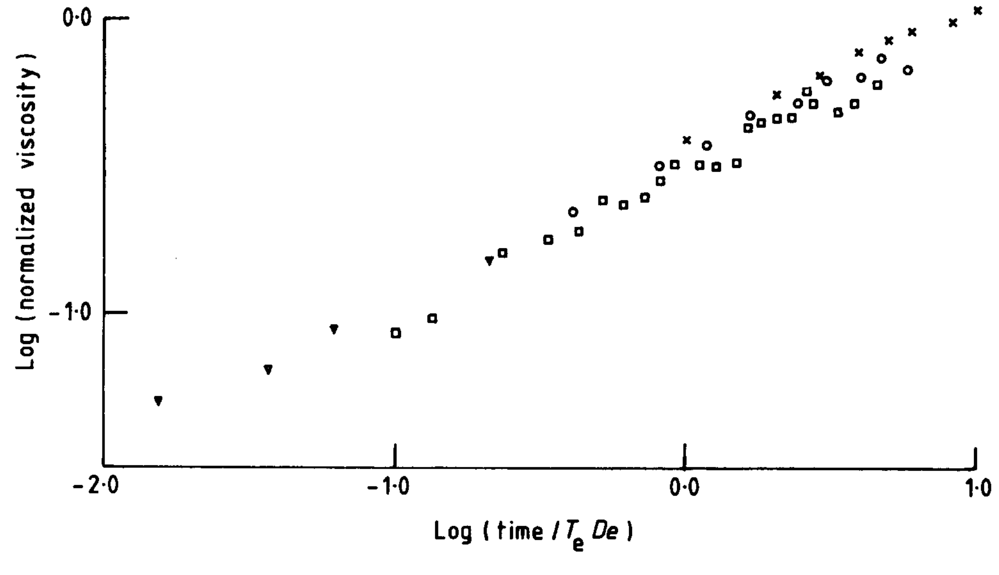

Fig. 2. Viscosity transients for OS138, normalized as described in the text, in response to various step changes in temperature. Temperature before steps was −19·0°C. Temperature after step and final equilibrium viscosity as follows: × : −26°C, 2×10^{12} Pa s: ○: −28°C, 7×10^{12} Pa s; □: −30°C, $3{\cdot}5 \times 10^{13}$ Pa s; ∇: −32°C, $1{\cdot}6 \times 10^{14}$ Pa s

The viscosity transients for both liquids show the behaviour expected from the simple model described earlier. The power ξ in equation (4), relating the viscosity and time during the transient, was the same for all the temperature steps used. This will probably not be true for very large steps in temperature or pressure as ξ depends on temperature and volume through the equation of state. The results for OS138 are shown in Fig. 2, and those for polystyrene 580 MW in Fig. 3, where the viscosities have been normalized by the final equilibrium viscosity after the temperature step and completion of the volume relaxation, and the time is normalized by the Deborah Number as described earlier.

The value for ξ for OS138 can be calculated using experimental data for a, b, and n. Previous data suggests $a = 2{\cdot}5 \times 10^{10}$(11), $b = 2{\cdot}5 \times 10^{-8}$(11)(12), and

$$n = 7{\cdot}5 \times 10^{-4}(7).$$

This suggests that $\xi = 0{\cdot}47$, in close agreement with the observed value of 0·5. Insufficient data are available for an estimate of the value of ξ for Polystyrene 580 MW to be made. The observed value is lower than for OS138, which could be due to a smaller coefficient of thermal expansion (13)(14).

For a fluid which has ξ in equation (4) equal to one the viscosity is given by $\mu = Gt$, so that the stress under constant shearing is given by $\sigma = G\dot{\gamma}t$, which is the short time approximation for the response of a Maxwell fluid to a step in shear rate. For values of ξ different to one the response can be approximated reasonably well by an exponential response with the values for the final viscosity and relaxation time being estimated less accurately from the short time data. The best exponential approximations to the experimental curves for OS138 are shown in Table 1, together with the Maxwell relaxation times. The close agreement between the fitted curves and those expected for shear viscoelasticity suggests that the two processes will be difficult to distinguish, except from an exact knowledge of the pressure and shear rate profiles.

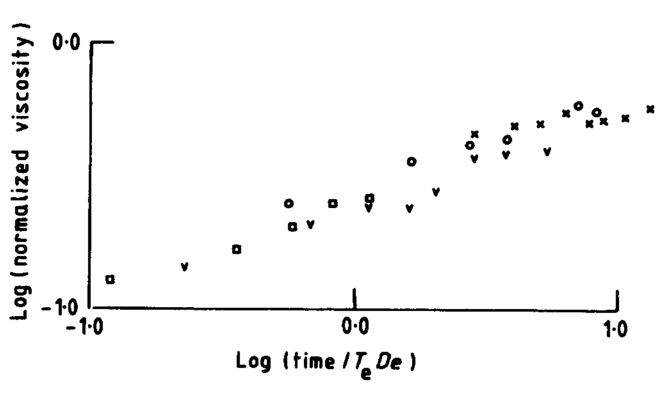

Fig. 3. Viscosity transients for Polystyrene 580 M.W., normalized as described in the text, in response to various step changes in temperature. Temperature before steps was −19·0°C. Temperature after step and final equilibrium viscosity as follows: × : −24°C, $4{\cdot}2 \times 10^{11}$ Pa s; ○: −26°C, $1{\cdot}8 \times 10^{12}$ Pa s; □: −28°C, $8{\cdot}5 \times 10^{12}$ Pa s; ∇: −30°C, $2{\cdot}7 \times 10^{13}$ Pa s

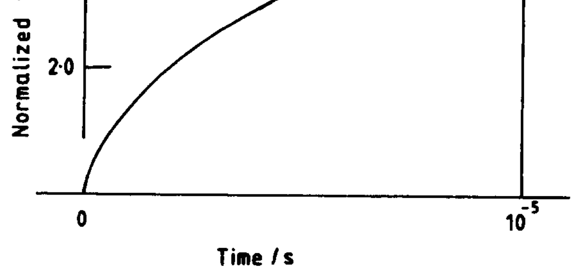

Fig. 4. Variation of viscosity for OS138 as it passes through a hypothetical EHD contact as specified in the text. This is for a square pressure profile and represents an upper limit on the viscosity in a real contact.

Table 1

Comparison of parameters calculated using exponential approximation to the viscosity transients with the actual values obtained, for OS138. $G = 10^9$ Pa. *Viscosity in* Pa s.

	Equilibrium viscosity		Relaxation time	
Temperature	actual	exp. approx.	observed	μ_∞/G
26	$2{\cdot}0 \times 10^{12}$	$2{\cdot}4 \times 10^{12}$	$8{\cdot}0 \times 10^{3}$	$2{\cdot}5 \times 10^{3}$
28	$7{\cdot}5 \times 10^{12}$	$5{\cdot}9 \times 10^{12}$	$1{\cdot}5 \times 10^{4}$	$5{\cdot}9 \times 10^{3}$
30	$3{\cdot}5 \times 10^{13}$	$2{\cdot}3 \times 10^{13}$	$4{\cdot}0 \times 10^{4}$	$2{\cdot}3 \times 10^{4}$
32	$1{\cdot}6 \times 10^{14}$	$1{\cdot}0 \times 10^{14}$	$2{\cdot}0 \times 10^{5}$	$1{\cdot}0 \times 10^{5}$

The linear approximation for the viscosity variation can be easily included in the Maxwell Model to show the effect of both processes operating at once. For the Maxwell Model with volume relaxation $\dot{\sigma} + \sigma/t = G\dot{\gamma}$, which has the solution $\sigma = G\dot{\gamma}t/2$. Thus the effect of both processes operating is to halve the rate of increase of the stress, compared with either process separately.

The Deborah Numbers of the experiments range from 10^{-1} to 5 for OS138 and 5×10^{-2} to 0·1 for polystyrene 580 MW. (The value of G for polystyrene is $1{\cdot}2 \times 10^9$ and for OS138 is 10^9Pa(**15**)). The following approximation to an EHD contact gives a Deborah number of 5×10^3, which means that in this situation the viscosity will be further from equilibrium than in the experiments described:

1. the pressure profile is a step to 5×10^8Pa,
2. operating with OS138 at 120°C,
3. thermal equilibrium,
4. residence time of 10^{-5} s.

The final equilibrium viscosity can be calculated (**11**) as $4{\cdot}8 \times 10^7$Pa s leading to the above quoted Deborah Number. The viscosity as a function of time is given by

$$\mu/\mu_\infty = 0{\cdot}35(t/4{\cdot}8 \times 10^{-2})^{0{\cdot}5}.$$

In 10^{-5} s as the fluid leaves the contact zone the viscosity has risen to about 5×10^{-3} of its final value. The mean viscosity in the contact region can be calculated by integrating the viscosity function over the time it is in the contact. For OS138 this gives $\mu = 0{\cdot}233(TG/\mu_\infty)^{0{\cdot}5}\mu_\infty$ for the region where the volume is relaxing as assumed in the theory. This is shown in Fig. 5 as a function of pressure.

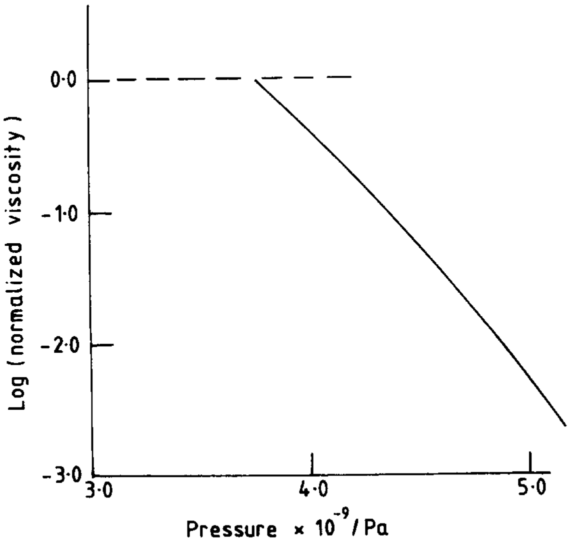

Fig. 5. Variation of viscosity of lubricant as it leaves the contact region as a function of pressure in the contact; all other conditions as for Fig. 4.

CONCLUSIONS

The results presented here show that in experiments where the Deborah Number is smaller than expected in EHD volume viscoelasticity plays an important role in determining the stress. It is thus possible to say that similar relaxation effects must occur in EHD. Approximations to the results obtained experimentally suggest that volume and shear relaxation effects will be difficult to distinguish.

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REFERENCES

(1) CROOK, A. W. 'The Lubrication of Rollers' *Phil. Trans. Roy. Soc.* 1963, **A255**, 281
(2) JOHNSON, K. L. and ROBERTS, A. D. 'Viscoelastic behaviour of an Elastohydrodynamic Lubricant Film' *Proc. Roy. Soc.* 1974, **A337**, 217
(3) HIRST, W. and MOORE, A. J. 'Elastohydrodynamic Lubrication at high pressures' *Proc. Roy. Soc.* 1978, **A360**, 403
(4) HARRISON, G. and TRACHMAN, E. G. 'The role of compressional viscoelasticity in the Lubrication of Rolling Contacts' *Trans. A.S.M.E.* 1972, 306
(5) FEIN, R. S. 'Possible role of compressional viscoelasticity in concentrated contact lubrication' *Trans. A.S.M.E.* 1967, 127
(6) KOVACS, A. J. 'Bulk creep and recovery in systems with viscosity dependent on free volume' *Trans. Soc. Rheol.* 1961, 285
(7) EASTWOOD, A. R. To be published in *J. Chem. Soc. Faraday Transactions*
(8) WALTERS, K. 'Rheometry' p. 200, Chapman and Hall, London
(9) JOHNSON, K. L. and TEVAARWERK, J. L. 'Shear behaviour of elastohydrodynamic oil films' *Proc. Roy. Soc.* 1977, **A356**, 215
(10) PLAZEK, D. J. 'A magnetic bearing torsional creep apparatus' *J. polym. Sci.* 1968, **6**, 621
(11) EASTWOOD, A. R. 'The viscous properties of liquids under extreme conditions of pressure' Symposium on Transport Properties of Fluids, National Engineering Laboratory, East Kilbride, Glasgow, April 1979
(12) McLACHLAN, R. *Ph.D. Thesis* Glasgow University
(13) GRAY, R. W., HARRISON, G. and LAMB, J. 'Dynamic behaviour of low molecular mass polystyrene melts' *Proc. Roy. Soc.* 1977, **A356**, 77
(14) BARLOW, A. J., ERGINSAV, A. and LAMB, J. 'Viscoelastic relaxation in supercooled liquids' *Proc. Roy. Soc.* 1967, **A298**, 481
(15) COCHRANE, J. Glasgow University, Private communication